

*Please do not simplify answers!*

1) Seven guests arrive at Alice's house for a dinner party. However, Alice only has seating for three people (One by the window, one under the air conditioning vent, and the other in a chair that is missing a cushion). Being a very controlling host, Alice will select which guest will sit where. How many ways can Alice select the seating arrangement? (Alice herself will not sit, and is not one of the guests)

2) At her job, Alice happens to be a password manager. She has decided that all company employees must abide by the following password regulations:

- Each password must be 8 or 9 characters.
- The first character must be a digit.
- The second, third, and fourth characters must be lowercase letters.
- The fifth character must be a 4.
- The sixth and seventh characters can be any lowercase digit, or a number.
- The eighth and [optionally] ninth characters must be one of the 6 symbols: !, @, #, \$, %, ^.

How many different passwords are there?

3) How many integer solutions are there to:

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 = 50$$

where  $x_i \geq 0$  and  $x_5 \leq 45$ .